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ANDREW McKELLAR, 1910–1960

By C. S. Beals
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Dr. Andrew McKellar was born February 2, 1910, in Vancouver, B.C., of Scottish parents who came to Canada in the first decade of the present century. His father John H. McKellar and his mother Mary Littleson McKellar came from the west coast of Scotland where Gaelic was still spoken in many homes and his childhood was inevitably influenced by a Scottish atmosphere. There were six in the family, Andrew, Neil, David, Jean, Janet and Isobel.

The quality of human warmth which his friends now remember with so much emotion, showed itself early, both in his family relationships and the friendships which he formed at school and university. During his student years he took part in many extra-curriculum activities but not to the detriment of his studies and the records show that he consistently maintained a brilliant academic standing from primary school to graduation from university. This high standing was not confined to science subjects, but included everything, Latin, English, history as well as mathematics and physics. All were firmly grasped and retained, in preparation for a future which all indications suggested would be a distinguished one.

Graduating with first-class honours in physics and mathematics from the University of British Columbia in 1930 he spent a summer with the Dominion Astrophysical Observatory in Victoria, B.C., before taking up graduate studies at the University of California where he received the degree of M.A. in 1932 and Ph.D. in 1933.

His thesis work at the University of California was concerned with the mass ratios and abundances of boron, lithium and silicon isotopes as determined by the methods of optical spectroscopy. After graduation from the University of California, Dr. McKellar felt that additional academic study and experience would stand him in good stead and so he applied for and won a United States National Research Council fellowship for two years of Post Doctorate study at the Massachusetts Institute of Technology. Here his work was concerned with the spectra of complex organic compounds and a study of the structure of the solar corona. Five publications appeared as evidence of his successful industry during his five years of post graduate training but an even greater tribute to his growing stature as a scientist was the deep respect and affection which he won from senior professors of these two distinguished institutions of learning.

During his graduate studies the summers of 1930, 1931 and 1932 were spent in astrophysical work at Victoria with J. S. Plaskett, W. E. Harper, H. H. Plaskett, J. A. Pearce, C. S. Beals and R. M. Petrie. When the time came to choose a life work the influence of this astronomical experience proved to be decisive. He joined the staff of the Dominion Astrophysical Observatory in 1935 and henceforth applied his keen scientific mind and his splendid academic training to the solution of astrophysical problems. His work at the Observatory covered such a wide range in the field of astronomical spectroscopy that it could only be fully appreciated in connection with a detailed bibliography of over 70 titles.*

*This will appear in the next issue of this JOURNAL.

It is not surprising that his early preoccupation with molecular spectra from a purely physical standpoint was reflected in his work in astronomy; he rapidly became a world authority on the applications of molecular spectroscopy to astronomical investigations. His work in this connection comprised not only the identification of molecules and free radicals in late-type stars, comets and interstellar space, but also the use of the peculiar properties of molecular spectra in the study of physical mechanisms in astronomical systems. Probably his most important contribution was his measurement of the relative abundance ratio of the carbon isotopes in stellar spectra. He showed that in most carbon stars the C^{12}/C^{13} ratio is about 5:1 instead of 90:1 as in terrestrial carbon samples; this constituted an observational proof of the existence of the "carbon-nitrogen cycle" which has been postulated as the source of energy in these stars. In 1940 he demonstrated the existence of CH, CN and NaH molecules in interstellar space by recognizing how the familiar laboratory spectra of these molecules would be altered by the extreme temperature conditions prevailing in the space between the stars. Special mention might also be made of his demonstration by a shrewd analysis of the intensity distribution in the molecular spectra of comets that solar radiation is the exciting mechanism of cometary emission.

While probably no scientist has been as successful as McKellar in applying the methods of molecular spectroscopy in astronomy, few have equalled his insight in applying the principles of line spectroscopy to astrophysical problems. His interests here ranged over a wide field including studies of the physics of novae and explorations of the atmospheres of giant stars. Much of his effort during the last decade was devoted to chromospheric studies of giant stars from spectra obtained during the ingress and egress of eclipsing binaries. His studies of certain eclipsing binaries revealed in a clear and direct way the existence of large turbulent motions in the extended atmospheres of cool giant stars.

His background in laboratory physics was undoubtedly responsible for his great interest in the design and application of new instrumental methods in astrophysics, and his contributions in the field of instrumental design have been of the greatest value not only in the observatory at Victoria but to others who sought his help and advice. Of comparable significance to his published work has been his influence on and help given to his colleagues, particularly younger scientists who turned naturally to him for advice and with whom he was always on terms of sympathetic understanding.

During the war of 1939–45 Dr. McKellar served with the Royal Canadian Navy in the Directorate of Operational Research and received the M.B.E. for his war services. In 1952–53 he occupied the post of

visiting Professor at the Physics Department, University of Toronto. He was a Fellow of the Royal Society of Canada, President of the Royal Astronomical Society of Canada 1959–60 and President of the Astronomical Society of the Pacific 1956–58.

Apart from his scientific activities, Dr. McKellar's outstanding personality and lovable personal qualities led to a wide circle of friends, both within and outside his profession. He was married in 1938 to Mary Crouch of Victoria and two children were born of the marriage, Robert and Barbara. The happy and informal atmosphere of their home made it a very pleasant place for their friends to gather, and an evening with the McKellars was always a memorable event.

Perhaps Andrew McKellar's greatest achievement was the manner in which he dealt with a severe health problem which he had to face during the last fifteen years of his life. Turning out much of his best work under the shadow of an incurable illness which eventually carried him away, he won the admiration of colleagues and friends throughout the world by his determination that his illness should not interfere with his scientific output, his work in astronomical organizations or his social and personal relationships. Evidence for the success of his efforts is to be found in the memories and hearts of his friends and by the fact that his last day's work at the Dominion Astrophysical Observatory was completed on May 2, 1960, only four days before his death.

THE REVERSING RAPIDS AT SAINT JOHN, NEW BRUNSWICK

By F. Keith Dalton

Toronto Centre

For many decades, the tidal phenomena and famous tourist attraction, near the mouth of the St. John River, have been known as "The Reversing Falls." There is not a waterfall, however, but a powerful tumbling rapids on the normal downstream flow of the river followed by a very strong upstream current when the rising tides take control and reverse the direction of movement of the water.

In an earlier issue of the Journal, (vol. 45, p. 225, 1951) under the title: "Fundy's Prodigious Tides and Petitcodiac's Tidal Bore", the writer explained the origin and described the nature and behaviour of the phenomenally high tides in the Bay of Fundy and the Minas Basin, and their effects on the shores and in the tributary rivers. The tides in the Basin are the highest in the world, 53½ feet, with nearly as high tides at the upper end of the Bay, so the conditions produced in the rivers are extreme.

In brief, the tides which affect the Bay of Fundy, and the rivers which discharge into it, commence in the southern Indian Ocean, west of Australia, and move westward increasing in magnitude as they travel until, passing around the Cape of Good Hope at the south end of Africa, they turn northward and proceed up the Atlantic Ocean. They reach their high levels on New Brunswick and Nova Scotia shores about twelve hours after the moon has crossed the local meridians. In the Bay of Fundy the tides are anomalistic—varying chiefly with the distance of the moon rather than with its phases and declination.

The Bay opens to the south so receives the tides directly as they rush northward in the ocean. As there is a resonant condition in water movement along this part of the North American shore, with nodes near Nantucket Island, very much higher tides are built up at the northern resonant loops which occur in this Bay. Moreover, the Bay becomes narrower and shallower toward its upper end causing the waters to build up to most unusual heights in the upper reaches and the tides rush on, still increasing, into adjoining Chignecto Bay and the Minas Basin where they attain maximum heights.

The St. John River, about 450 miles in length—the longest river on the North Atlantic seaboard with the exception of the St. Lawrence—flows into the Bay of Fundy at the city of Saint John, New Brunswick. At this

point, Fundy's tides rise to a height of 28½ feet and reverse the flow in this river for a considerable distance upstream from its mouth creating fresh-water tides farther up by pushing back the normal river waters. While every river or brook that flows into the Bay of Fundy, the Minas Basin or associated bays, suffers reversal of flow when tides rise, any of these streams which happens normally to have rapids near its mouth will also have a so-called "reversing rapids". The chief claim to fame then of the phenomena at Saint John is based upon the enormous amount of river water involved and the great strength shown by the rising tides in overcoming the flow. It is a very spectacular sight and well worth a visit to observe.

The Rapids. During the normal downstream flow of the St. John River, during a long period under low-tide conditions, there is about one half mile of vigorous turbulent Rapids over a rocky bed, through a narrow cliff-walled gorge and under the railway and highway bridges into the harbour. (See figures 1 and 2.)

The overall possible difference in level between the brink and tail of these Rapids is about 14½ feet. As the rise of Fundy's tides is about twice this amount, the tide waters can and do completely overcome the Rapids twice each day.

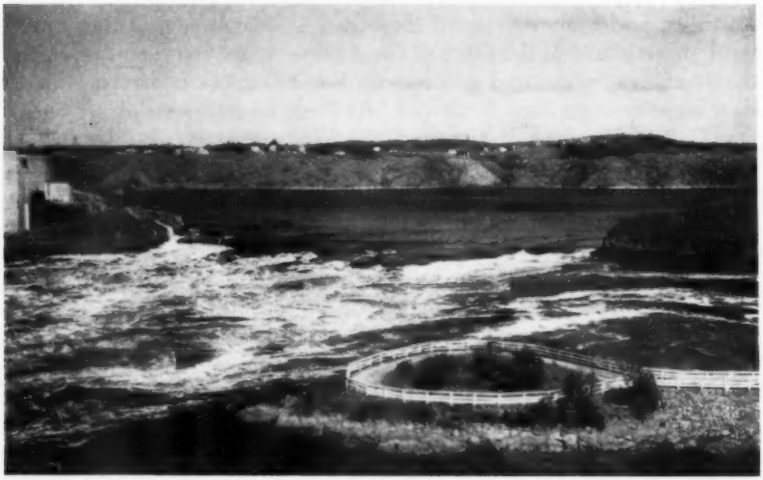

FIG. 1—The brink of the Rapids and turbulent downstream flow of river waters at low tide. Several observation areas are provided for visitors. (Courtesy of the Saint John Municipal Tourist Office.)

FIG. 2—The discharge of the Rapids at low tide—under the railway and highway bridges and through the gorge into the harbour. (Courtesy of the Saint John Municipal Tourist Office.)

While tide levels rise, the tide waters creep up the Rapids, with normal river flow continuing downstream and over the top of the incoming tides. There is no really clear demarcation while the tides overcome the Rapids —until about the time that the rising levels reach the brink. In the same way, while the tides are ebbing, there is no definite line or sudden change where the receding tide waters are leaving the Rapids.

Slack Waters. In tidal streams, slack water occurs when the flow stops in one direction and is about to start in the opposite direction. There are two such cessations of flow in each complete cycle of events but these very rarely occur at the same times as high and low tides. There is usually some delay.

When the rising tides, creeping up the Rapids, reach the brink, the cessation of flow is known as "Low Slack Water." This occurs about 3 hours 50 minutes after low tide in the harbour.

The flow reversal above the Rapids, over the brink, now begins—and this upstream movement lasts for about 4 hours 40 minutes. By that time, tides are ebbing in the harbour and in this part of Fundy so again, and this time at highest level, there is the second cessation of movement and the flow is about to start in the downstream normal direction (see figure

FIG. 3—Upstream end of the Rapids area at high slack water when the tides have stopped the river flow—and navigation over this area is possible. The white area of the water is quietly floating foam. (Courtesy of the Saint John Municipal Tourist Office.)

3). This stage is known as "High Slack Water" and occurs about 2 hours 25 minutes after high tide in the harbour.

The waters now have flooded the Rapids area so only eddies and ripples in the stream are to be seen. Under these high-water conditions, navigation past the Rapids by small craft is possible and safe for about 45 minutes, and is quite customary (see figure 4). These boats take advantage of the upstream flow, just before high slack, to move in that direction but hasten back down stream with the outgoing tide before the water over the brink of the Rapids becomes too shallow for safe passage. Those who navigate this area must keep close check on the times at which the various phases of these phenomena occur each day—and there is a complete cycle of events every 12½ hours but one hour later on each successive day. To ignore the tidal movements and the tide tables could easily result in a shipwreck.

For a half hour or more after high slack water, there is downstream flow but no evidence yet of rapids for the water level over the brink is still too high, the water too deep. Then the Rapids appear and increase gradually in length and more vigorous flow. This impressive sight lasts until the tides creep up over the Rapids again to reach the brink at the next low slack water, a period of about six hours.

FIG. 4—Downstream end of the Rapids area at high slack water with two small vessels, a tug and a tanker, moving upstream through the gorge and under the bridges—viewed from an observation area beside the Tourist Centre. (Courtesy of the Saint John Municipal Tourist Office.)

Varying Conditions. The lengths of the periods of upstream flow, from low slack to high slack, and of evident downstream rapids, vary with the amount of rise of the tides in the Bay of Fundy—from 21 feet, minimum, to 28½ feet, maximum. The amount of water flowing over the Rapids, however, varies considerably with river flood and dry season conditions as well as with the amount of tide water forced up the river. The greatest discharge over the brink of the Rapids, as far as tidal conditions alone are concerned, occurs just after the brink appears, nearly an hour after high slack water. With the large amount of outgoing water, the river level now is dropping so, with lowering head of water, the discharge over the Rapids gradually decreases until the next low slack occurs. Precipitation, of course, will affect the river flow in any season.

The matter of power development at these Rapids has, undoubtedly, been given serious thought but the periodic reversals of flow of water and particularly the slack phases, when there is no available operating head, present a difficult problem. Nevertheless, the tidal waters of Passamaquoddy Bay, along the Fundy shore and about 50 miles west of the city of Saint John, are now being carefully considered for development and, if the problem can be solved satisfactorily there, the Reversing

Rapids may be harnessed later. However, generating stations that depend upon tidal movements can not be operated on isolated power systems; they must be connected to other stations which provide steady power. The Province of New Brunswick already has a suitable power system which could absorb the periodic output from tidal generating stations, and make full use of it, so success in such development may be realized there in the near future.

Observing the Rapids at Saint John. The Reversing Rapids are astronomical phenomena of decided interest to both professional and amateur astronomers for they demonstrate the gravitational effect of the sun and moon on the oceans of our earth, and present the results at very close range. These Rapids also form a very fine tourist attraction for the city of Saint John and are conveniently placed for observation.

Unlike the tidal bore at Moncton, about 90 miles away to the east, the Reversing Rapids are not just a short-time entertainment, lasting only a few minutes twice a day. The whole cycle of events at the Rapids spreads over 12½ hours, and these cycles occur about an hour later on each successive day. There are nearly two complete cycles every day.

Probably the best time for the observer to arrive at the Rapids is about halfway between low slack and the next high slack water. He then will see first the reversed upstream flow of water gradually decreasing over the next two hours, until standstill at highest level. He can now watch the flow start downstream again and become faster. In about an hour, the brink of the Rapids will begin to form and the waters increase in turbulence. The development of the Rapids is most interesting but, while the tail edge of the retreating tide is not well defined, one is aware of the gradual increasing and lengthening of the turbulent area until the full flow of the Rapids is exposed. This will require about three hours following the appearance of the brink. The astronomer or tourist, therefore, should count upon spending about six hours at the Rapids area in order to see all of the chief phases and thus get the best impression of the phenomena. He may, of course, slip away for short periods to visit the central part of the city, only a mile distant, provided he choose carefully just when to be absent so as not to miss the important events at the Rapids.

To aid the visitor and observer, Tide Tables for the Rapids are published every summer by the Municipal Tourist Development Office at Saint John and are distributed to all who desire them—either sent by mail or handed to the tourist when he arrives in the city. There are several observation areas provided and well placed for good views of

the Rapids. As a further aid to ensure that the visitor shall have a continuous view of the phenomena, the Rapids are illuminated every night. This is very striking for there is so much foam moving with the water that the eddies and turbulence are clearly outlined by the illumination.

In observing phenomena such as the Reversing Rapids at Saint John, one is likely to find a few surprises for, although all aspects of the various phases have been calculated closely and predicted for him in both time of occurrence and rise of the tides, he often finds some feature which he has not just expected. Such surprises add greatly to the interest of the visit.

While the Tide Tables give the times of high and low tides in the harbour, and of high and low slack waters at the brink of the Rapids, they do not attempt to explain the strange feeling of nervous tension that one experiences as he watches the strong upstream current during reversal of river flow—as though it must lead to some dire consequences—and then realizes the sudden release of tension as the flow stops, starts downstream to become normal again.

The Reversing Rapids are well worth a visit and the spending of six or seven hours to watch, whether the observer be interested in astronomy or be an inquisitive tourist.

THE ORBIT OF THE SPECTROSCOPIC BINARY H.D. 174369

By William Wehlau
University of Western Ontario, London, Ontario

The star H.D. 174369, α (1900) $18^h\ 45.1^m$, δ (1900) $+ 24°56'$, vis. mag. 6.56, spectral type A2n, was announced to be a spectroscopic binary in 1945 at the David Dunlap Observatory (Young 1945). In 1955 there were 29 plates available for a determination of the orbit. Additional information was required to resolve an uncertainty in the period. In 1957 and 1958 eleven more plates were obtained for this purpose. All plates were taken with the 25-inch camera of the one-prism spectrograph at the Cassegrain focus of the 74-inch reflector at the David Dunlap Observatory, giving a dispersion of about 33 A./mm. at Hγ. The lines used for the radial velocity determination were selected from among the following: 3933.664, 4045.739, 4063.539, 4071.687, 4077.644, 4143,682, 4226.911, 4233.275, 4250.481, 4271.548, 4282.621, 4307.892, 4315.786, 4325.674, 4340.466, 4351.839, 4443.578, 4468.643, 4481.310, 4549.550. The diffuse nature of the lines made it impossible to measure all of those listed for each spectrum. However $\lambda\lambda$ 4340 and 4481 were measured on

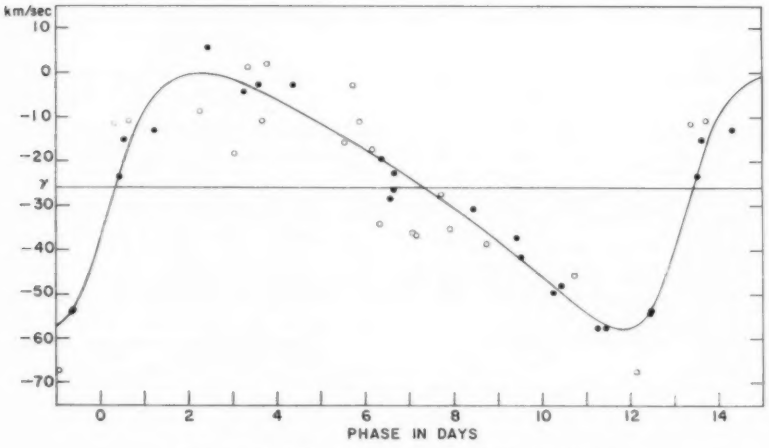

Fig. 1—Radial-velocity curve for H.D. 174369.

almost every plate, and in no case were fewer than four lines used. The data for the plates are listed in Table I.

The plates cover an interval of time long relative to the adopted period. The values of the radial velocities for 1958, six in number, appear to fall somewhat low relative to the adopted velocity curve, although the poor lines in the spectra of this star make it difficult to determine whether or not this is significant. For these reasons, it was decided to use only twenty plates of good quality taken during 1955 for the determination

TABLE I

OBSERVATIONS OF H.D. 174369

J. D. 243	V_0 km./sec.	No. of lines	Phase from final T	V_c km./sec.	$V_0 - V_c$ km./sec.
0602.594	−8.9	9	2.263	−0.1	−8.8
0616.748	+1.3	11	3.336	−3.0	4.3
0809.928	−11.5	16	.301	−27.3	15.8
1003.486	−45.6	10	10.725	−52.0	6.4
1314.619	−35.3	12	7.914	−29.8	−5.5
2008.727	−38.7	13	8.729	−35.9	−2.8
2340.860	−10.8	10	.756	−13.8	3.0
2476.476	−15.8	8	5.562	−14.7	−1.1
3463.744	(−26.0)	8	11.755	−58.0	(32.0)
3468.657	−2.9	10	3.587	−4.1	1.2
3471.654	−28.7	8	6.584	−20.9	−7.8
3471.744	−22.9	12	6.674	−21.5	−1.4
3478.690	−15.1	7	.539	−19.9	4.8
3484.819	−26.3	7	6.668	−21.5	−4.8
3487.662	−41.6	14	9.511	−42.0	0.4
3489.590	−57.4	11	11.439	−57.0	−0.4
3490.642	−53.8	9	12.491	−52.7	−1.1
3491.653	−23.3	8	.421	−23.5	0.2
3499.689	−30.7	12	8.457	−33.9	3.2
3500.644	−37.1	11	9.412	−41.2	4.1
3501.662	−48.0	9	10.430	−49.6	1.6
3503.665	−54.2	9	12.433	−53.7	−0.5
3506.735	+5.8	11	2.422	−0.3	6.1
3507.568	−4.2	9	3.255	−2.7	−1.5
3508.689	−2.7	10	4.376	−8.1	5.4
3510.693	−19.3	5	6.380	−19.6	0.3
3514.572	−49.5	7	10.258	−48.3	−1.2
3515.583	−57.5	6	11.270	−56.0	−1.5
3518.624	−13.0	8	1.230	−5.2	−7.8
6100.524	−17.5	5	6.173	−18.3	0.8
6100.684	−34.1	10	6.334	−19.3	−14.8
6114.515	−36.2	8	7.083	−24.3	−11.9
6114.603	−36.7	6	7.171	−24.9	−11.8
6119.551	−67.4	11	12.119	−57.2	−10.2
6123.528	−18.3	10	3.015	−1.8	−16.5
6361.698	−2.9	5	5.727	−15.7	12.8
6361.846	−11.0	4	5.875	−16.5	5.5
6372.710	−10.9	4	3.658	−4.4	−6.5
6372.845	+2.1	4	3.793	−5.1	7.2
6376.769	−27.4	8	7.717	−28.4	1.0

of the elements. One other plate taken that year, J.D. 243,3463.744, was omitted because the very weak spectrum gave inconsistent results for different lines and on successive measurements. Tanner's method (1948) was used to check for the possibility of an associated short-period variation and no indication of one was found.

The twenty observations were grouped to form eleven normal points. Preliminary elements were determined using Irwin's method (1952). A least-squares solution was made for K, γ, e, ω, T. The preliminary and final elements are given in Table II, together with the mean errors ob-

TABLE II

ORBITAL ELEMENTS FOR H.D. 174369

Element		Preliminary	Final M.E.
Period	P	13.081 days	13.081 days
Eccentricity	e	0.40	0.391 ± 0.049
Angle of periastron	ω	255°	253.5 ± 10.8
Periastron passage	T	J.D. 2433504.3	2433504.313 ± 0.289
Velocity of system	γ	−24.24 km./sec.	-25.86 ± 2.64
Semi-amplitude	K	31.5 km./sec.	28.94 ± 2.03
$a \sin i$			4.79×10^6 km.
Mass function			$0.0256 \odot$

tained from the least-squares solution. The individual observations and the least-squares solution are plotted in figure 1. The filled circles represent observations used in the least-squares solution and the open circles represent the other observations.

It is a pleasure to express my appreciation to Dr. John F. Heard and Miss Ruth Northcott of the David Dunlap Observatory for the loan of the plates and for obtaining others as required. Financial support was supplied by the National Research Council of Canada.

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ELEMENTS OF ASTROMECHANICS

By Peter van de Kamp
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IV. Keplerian Motion. The Time Element in Orbits

1. *Elliptical orbit; Kepler's problem*

The location in an elliptical orbit is determined by the three *dynamical elements*: period P, eccentricity e, time of perihelion passage T, and the epoch of observation t. This is called Kepler's problem and the solution is easily followed by introducing Kepler's *auxiliary circle*, which is located in the orbital plane tangent to the orbit in perihelion and aphelion and from which the elliptical orbit may be derived by the foreshortening factor, $\sqrt{1 - e^2}$ perpendicular to the major axis (Chapter I). The radius of Kepler's circle is therefore equal to the semi-axis major a of the orbit.

The angular co-ordinate of the position in the ellipse is the *true anomaly* v counted from perihelion in the direction of orbital motion. The position of the corresponding location in the auxiliary circle is measured at the centre by the *eccentric anomaly*, E, also counted from perihelion in the direction of orbital motion. It is convenient to introduce the *unit orbit*

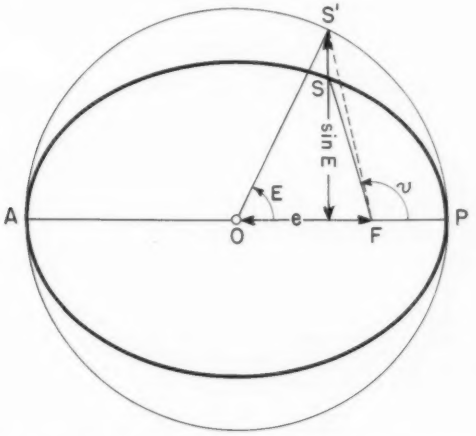

Fig. 4.1—Kepler's problem. Relation between position S on unit orbit and corresponding position S' on auxiliary unit circle.

and the *unit circle*, for which $a = 1$. The radius vector FS is the projection of the radius vector FS', which sweeps over equal areas of the circle in equal intervals of times. The area of the unit circle amounts to π, and is swept over in one period P. Hence the area $FS'P$ in the unit circle, covered in the time $t - T$ since perihelion passage, amounts to $\pi(t - T)/P$. This area may be expressed as the difference between the sector $S'OP$ and the triangle $OS'F$, i.e.

$$S'OP - OS'F = FS'P,$$

or

$$\frac{E}{2} - \frac{e \sin E}{2} = \pi \frac{t - T}{P};$$

and

$$E - e \sin E = 2\pi \frac{t - T}{P}. \tag{4.1}$$

The right-hand side of the equation is also called the *mean anomaly* M, and thus we arrive at *Kepler's equation*

$$E - e \sin E = M, \tag{4.2}$$

a transcendental functional relation between E, e and M.

2. *Solution of Kepler's equation*

For any pair of values e and M this equation may be solved to any accuracy by development in series. For *small eccentricities* a high degree of approximation is obtained by first assuming $E = M$: i.e.

$$E = M + e \sin M. \tag{4.3}$$

A next approximation is the following:

$$\begin{aligned} E &= M + e \sin (M + e \sin E) \\ &= M + e \sin M \cos (e \sin E) + e \cos M \sin (e \sin E) \end{aligned}$$

or, since e is small

$$E = M + e \sin M + e^2 \cos M \sin M,$$

whence

$$E = M + e \sin M + \frac{e^2}{2} \sin 2M. \tag{4.4}$$

An elegant, general graphical solution was given by J. J. Waterson (*Monthly Notices*, vol. 10, p. 169, 1850). Note that

$$\sin E = \frac{1}{e} (E - M) \tag{4.5}$$

or also

$$e \sin E = E - M \tag{4.6}$$

Therefore, by plotting the ordinate sin E from $E = 0$ to π, a straight line of slope $1/e$ from the point M on the axis of abscissae will intersect the sine curve at a point whose abscissa is E. Or, a straight line of slope 1 (45°) may be used to intersect a plotted curve e sin E for a particular orbit, or a set of curves of e sin E may be used for values of e differing by say 0.1, and values of E interpolated. The same graph may be used between π and 2π by subtracting the values of M and E from 360°.

An illustration is given in section 5 (figure 4.3).

J. J. Åstrand's tables (Leipzig: Wilhelm Engelmann, 1890) give E as a function of M and e.

To obtain the position in the orbit from E, we proceed as follows: v may be calculated from

$$\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}, \tag{1.39}$$

and r from

$$r = a\,(1 - e \cos E),$$

and thus the position in the orbit is known.

It is convenient to introduce the *elliptical rectangular co-ordinates* x,y, in the *unit orbit*, given by

$$\begin{aligned} x &= \cos E - e \\ y &= \sin E \sqrt{1 - e^2}. \end{aligned} \tag{4.7}$$

Recall that

$$\begin{aligned} r \cos v &= a\,(\cos E - e), \\ r \sin v &= a \sin E \sqrt{1 - e^2}. \end{aligned} \tag{1.34}$$

The elliptical rectangular co-ordinates in the orbit of semi-axis major a are thus given by

$$\begin{aligned} r \cos v &= a\,x \\ r \sin v &= a\,y \end{aligned} \tag{4.8}$$

Tables published by the Union Observatory (*Circular*, no. 71, 1927) give x and y as functions of e and M so that E need not even be computed. These tables are primarily designed for the analysis of double-star orbits.

3. *Average values of the radius vector r and of its inverse value*

The average values of r and $1/r$ with respect to the time and true anomaly are of interest. These averages are, of course, functions of a and e, and both expressions

$$r = \frac{p}{1 + e \cos v} \tag{1.19}$$

and $$r = a(1 - e \cos E) \tag{1.35}$$

may be used to derive the average values. Moreover, the differential expression of Kepler's equation proves to be useful. From (4.1) we have

$$dE - e \cos E \, dE = \frac{2\pi}{P} dt, \tag{4.9}$$

and, after substituting (1.35)

$$\frac{dt}{dE} = \frac{r P}{2\pi a}. \tag{4.10}$$

Recall, further, that

$$\frac{dv}{dt} = \frac{2A}{r^2} \tag{1.3}$$

hence, substituting (4.10)

$$\frac{dv}{dE} = \frac{A P}{\pi a r} = \frac{\pi a b}{\pi a r} = \frac{b}{r}. \tag{4.11}$$

(a) The time-average of r is defined as

$$\bar{r}_t = \frac{\int_0^{P/2} r \, dt}{\int_0^{P/2} dt} = \frac{\int_0^{\pi} r \frac{dt}{dE} dE}{\frac{P}{2}}. \tag{4.12}$$

Substitution of (4.10) into (4.12) leads to

$$\bar{r}_t = \frac{\int_0^{\pi} r^2 \, dE}{\pi a}. \tag{4.13}$$

Substitution of (1.35) yields

$$\bar{r}_t = \frac{a}{\pi} \int_0^{\pi} (1 - e \cos E)^2 \, dE$$

$$= \frac{a}{\pi} \left[\int_0^{\pi} dE - 2e \int_0^{\pi} \cos E \, dE + e^2 \int^{\pi} \cos^2 E \, dE \right]$$

$$= \frac{a}{\pi} \left(\pi - 0 + e^2 \frac{\pi}{2} \right):$$

(note that

$$\int_0^{\pi} \sin^2 E \, dE = \int_0^{\pi} \cos^2 E \, dE, \text{ and also } \int_0^{\pi} (\sin^2 E + \cos^2 E) dE = \pi.)$$

Hence,
$$\bar{r}_t = a\left(1 + \frac{e^2}{2}\right). \tag{4.14}$$

(b) The time-average of $1/r$ is defined as

$$\left(\frac{\overline{1}}{r}\right)_t = \frac{\int_0^{P/2} \frac{1}{r}\,dt}{\int_0^{P/2} dt} = \frac{\int_0^{\pi} \frac{1}{r}\frac{dt}{dE}\,dE}{\frac{P}{2}}; \tag{4.15}$$

or, after substituting (4.10)

$$\left(\frac{\overline{1}}{r}\right)_t = \frac{\frac{P}{2\pi a}\int_0^{\pi} dE}{\frac{P}{2}}; \tag{4.16}$$

whence
$$\left(\frac{\overline{1}}{r}\right)_t = \frac{1}{a}. \tag{4.17}$$

(c) The value of r averaged over the true anomaly yields

$$\bar{r}_v = \frac{\int_0^{\pi} r\,dv}{\int_0^{\pi} dv} = \frac{\int_0^{\pi} r\frac{dv}{dE}\,dE}{\pi}. \tag{4.18}$$

Substitution of (4.11) gives

$$\bar{r}_v = \frac{b\int_0^{\pi} dE}{\pi}, \tag{4.19}$$

whence
$$\bar{r}_v = b = a\sqrt{1 - e^2}. \tag{4.20}$$

(d) The value of $1/r$ averaged over the true anomaly v yields

$$\left(\frac{\overline{1}}{r}\right)_v = \frac{\int_0^{\pi} \frac{1}{r}\,dv}{\int_0^{\pi} dv} = \frac{\int_0^{\pi} (1 + e\cos v)dv}{p\pi} =$$

$$= \frac{1}{p\pi}\left[\int_0^{\pi} dv + e\int_0^{\pi} \cos v\,dv\right] = \frac{1}{p\pi}(\pi + 0). \tag{4.21}$$

Hence,
$$\left(\frac{\overline{1}}{r}\right)_v = \frac{1}{p} = \frac{1}{a(1 - e^2)}. \tag{4.22}$$

To summarize,

$$\bar{r}_t = a\left(1 + \frac{e^2}{2}\right), \tag{4.14}$$

$$\left(\frac{1}{r}\right)_t = \frac{1}{a}, \tag{4.17}$$

$$\bar{r}_v = b = a\sqrt{1 - e^2}, \tag{4.20}$$

$$\left(\frac{1}{r}\right)_v = \frac{1}{p} = \frac{1}{a(1 - e^2)}. \tag{4.22}$$

4. *Parabolic orbit*

The equation in polar co-ordinates reduces to

$$r = \frac{p}{1 + e\cos v} = q\sec^2\frac{v}{2} \tag{4.23}$$

where $q = \frac{p}{2}$. (1.42), (1.43)

Now $A = \frac{rV_\theta}{2}$, (1.5), (2.3)

and $V_p^2 = \frac{2\mu}{r}$. (3.47)

At perihelion $r = q, \quad V_\theta = \sqrt{\frac{2\mu}{a}}$,

whence $A = \sqrt{\frac{q\mu}{2}}$. (4.24)

The general formula $A = \frac{r^2}{2}\frac{dv}{dt}$ (1.3)

may thus be written as $r^2 dv = \sqrt{2q\mu}\, dt$: (4.25)

or, substituting (4.23) $q^2\sec^4\frac{v}{2}\, dv = \sqrt{2q\mu}\, dt$, (4.26)

from which $\sec^4\frac{v}{2}\, dv = \sqrt{\frac{2\mu}{q^3}}\, dt$. (4.27)

We may write this as

$$\left(1 + \tan^2\frac{v}{2}\right)\left(1 + \tan^2\frac{v}{2}\right) dv = 2\left(1 + \tan^2\frac{v}{2}\right) d\tan\frac{v}{2} = \sqrt{\frac{2\mu}{q^3}}\, dt,$$

or $d\tan\frac{v}{2} + \tan^2\frac{v}{2}\, d\tan\frac{v}{2} = \sqrt{\frac{\mu}{2}}\frac{1}{q^{3/2}}\, dt$: (4.28)

whence
$$\tan\frac{v}{2}+\frac{\tan^3\frac{v}{2}}{3}=\sqrt{\frac{\mu}{2}}\frac{1}{q^{3/2}}(t-T). \tag{4.29}$$

For any specific orbit with inertial constant μ, perihelion distance q and perihelion passage T, we may calculate

$$\text{“}M\text{”}=\frac{t-T}{q^{3/2}}\sqrt{\frac{\mu}{2}} \tag{4.30}$$

and write
$$\tan\frac{v}{2}+\frac{\tan^3\frac{v}{2}}{3}=\text{“}M\text{”} \tag{4.31}$$

The calculation of v is facilitated by Barker's tables which relate the one real solution of v and the function “M” (Watson, J. C., *Theoretical Astronomy*, p. 60, also Table VI, p. 566, 1885).

5. *The orbit of the artificial satellite Explorer VI* (1959 δ2)

Kepler's problem will now be illustrated for the orbit of the artificial satellite Explorer VI, launched 1959 August 7. This orbit presents a good text-book case of an elliptic orbit with an appreciable eccentricity. (cf. also Ch. 2, fig. 2). The calculations that follow are based on data revised on October 26, 1959 and kindly supplied by Dr. Charles A. Whitney. See also “Anticipated Orbital Perturbations of Satellite 1959 Delta Two” by Yoshihide Kozai and Charles A. Whitney, Smithsonian Institution Astrophysical Observatory, *Research in Space Science*, Special Report, no. 39.

Explorer VI reached the greatest apogee distance of any artificial satellite launched so far. Its orbit is markedly affected by lunar perturbations. These in turn affect the perigee and increase air-drag. Without the moon the life expectancy of Explorer VI would be more than two decades; the perturbations reduce the life-expectancy to about two years.

The following dynamical elements are taken from a private communication from Charles A. Whitney, dated 1960 February 6.

P = 752.770 minutes = 45166.2 seconds (anomalistic period).
e = 0.758
T = 1960 February $3^d\ 8^h\ 55.57^m$ Universal Time.

The period decreases at the rate of 0.03774 minutes for each revolution. The size of the orbit is given by:

radius vector at perigee r_P = 6622.6 kilometres (4115.1 miles)
radius vector at apogee r_A = 48201.0 kilometres (29950.6 miles).

[1 (statute) mile = 1.60935 kilometres]

The apogee distance diminishes by 3.48 kilometres (2.16 miles) each day.

The semi-axis major has been taken as

$$a = 27411.8 \text{ kilometres} = 2.74118 \times 10^9 \text{cm. (17032.85 miles).}$$

The values of a and P are consistent with a value of 5.978×10^{27} grams for the mass M of an assumed spherical earth, calculated from

$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2} \tag{2.31}$$

neglecting the mass of the satellite.

The inertial constant

$$\mu = GM \tag{4.32}$$

in the c.g.s. system amounts to

$$\mu = 6.67 \times 10^{-8} \times 5.978 \times 10^{27} = 3.99 \times 10^{20}$$

for the gravitational field of an assumed spherical earth.

The relation for the linear orbital velocity

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \tag{3.45}$$

thus becomes $$V^2 = 3.99 \times 10^{20}\left(\frac{2}{r} - \frac{1}{a}\right)$$

from which the linear velocity may be calculated for any value of r. The velocities at perihelion and aphelion are given by

$$V_P^2 = \frac{\mu}{a}\frac{1+e}{1-e}, \tag{3.32}$$

and

$$V_A^2 = \frac{\mu}{a}\frac{1-e}{1+e}. \tag{3.33}$$

Since $$\frac{\mu}{a} = \frac{3.99 \times 10^{20}}{2.74118 \times 10^9} = 14.56 \times 10^{10},$$

and $$\frac{1+e}{1-e} = 7.26, \qquad \frac{1-e}{1+e} = 0.138,$$

it follows that $$V_P^2 = 10.57 \times 10^{12}$$

$$V_A^2 = 2.01 \times 10^{10};$$

whence

$$V_P = 1.028 \times 10^6 \text{ cm./sec.} = 10.28 \text{ km./sec. (23,000 m.p.h.).}$$

$$V_A = 1.42 \times 10^5 \text{ cm./sec.} = 1.42 \text{ km./sec. (3,180 m.p.h.).}$$

Figure 4.2 illustrates the orbital positions (in February 1960) for successive twelfths of one period of revolution, beginning with perigee. The graphical solution of Kepler's problem is obtained by plotting the

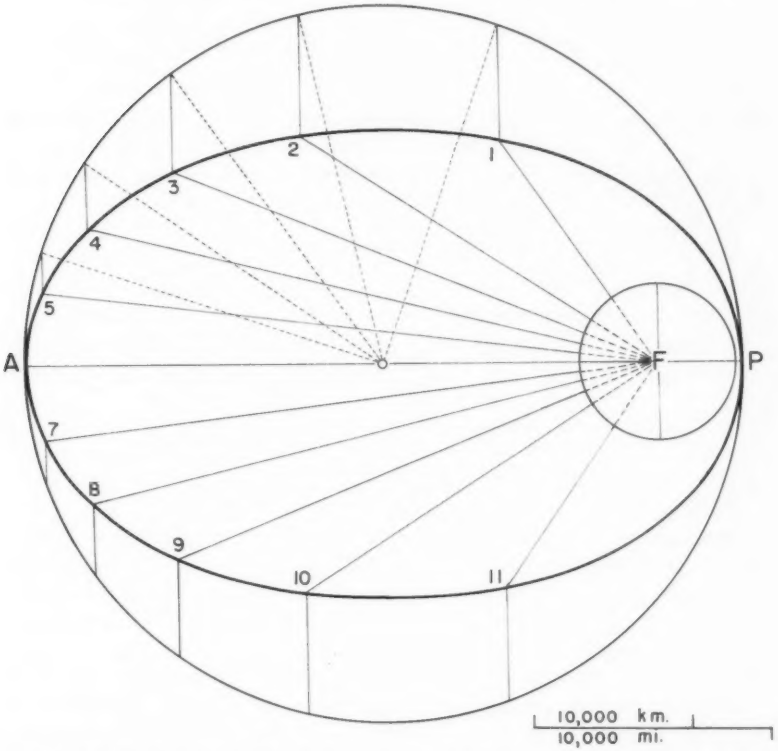

FIG. 4.2—Explorer VI (1958 δ2). Positions in orbit and auxiliary circle for successive twelfths of one period of revolution.

curve $e \sin E$ against E (figure 4.3). The resulting values of E for values of $M = 0, 30°, 60°$ etc., yield positions on the auxiliary circle from which the corresponding positions in the orbit are easily obtained by projection.

The values of M and E for one half of the orbit are tabulated below in addition to the values x and y of the elliptical rectangular co-ordinates taken from the tables in *Union Observatory Circular*, no. 71. The values of E from the graphical solution agree within one degree with those listed in Åstrand's tables.

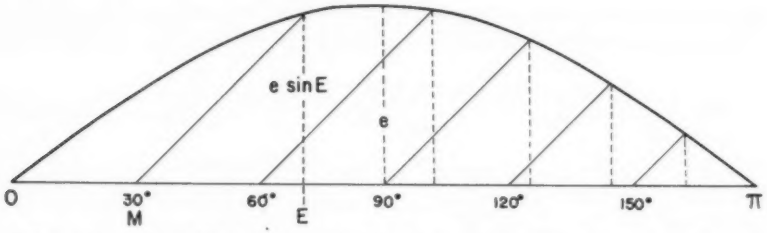

FIG. 4.3—Explorer VI. Graphical solution of Kepler's equation for values of M from 0° to 180°, at intervals of 30°.

$\frac{t-T}{P}$	M	E	x	y
0	0°	0°	+ .242	.000
1/12	30	71.1	− .434	+.617
2/12	60	102.4	− .973	+.637
3/12	90	125.4	−1.337	+.532
4/12	120	144.9	−1.577	+.375
4/12	150	162.8	−1.713	+.193
6/12	180	180.0	−1.758	.000

(*To be continued*)

CANADIAN SCIENTISTS REPORT—XXIII

INTERFERENCE FILTERS

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Introduction. The first interference filters were made about 20 years ago by Geffcken of the Zeiss firm, but they did not see much application until some time after the war. Their advantages over conventional absorption filters are that they can isolate a very narrow region of the spectrum and that this region can be at any desired wave-length. Along with these advantages go some limitations which must be understood if the filters are to be fully effective. First, the wave-length passed changes with angle of incidence so that they must be used with caution in a convergent beam. Second, the light which is not transmitted is reflected and may cause trouble. In the following we will discuss the action of the filter in physical terms and describe briefly how it is produced. Finally, some applications to astrophysics and upper-atmospheric physics will be discussed.

General. In principle an interference filter is identical with a Fabry-Perot interferometer. The difference is that the order of interference is very small instead of many thousands or even hundreds of thousands. Two identical reflecting layers are separated by a transparent layer, solid in the filter but usually air in the interferometer. The thickness of the spacer layer must be accurately constant over the whole area; in the filter this condition is attained by careful vacuum evaporation, but in the interferometer the glass surfaces carrying the reflecting layers must be flat to a very small fraction of a wave-length and held parallel by an accurate spacer. Let us consider figure 1 which shows such a system schematically. We assume that each reflecting layer reflects 95 per cent. of incident light and transmits the other 5 per cent.; the effect of deviations from this ideal behaviour will be considered later. Light of wave-length λ and intensity 100 is incident at right angles, and the optical thickness of the spacer layer is $\lambda/2$. Initially, most of the light is reflected, but some of it leaks into the filter and starts to build up a standing wave by multiple reflections back and forth. The intensity builds up because after two reflections the wave is joined by another transmitted

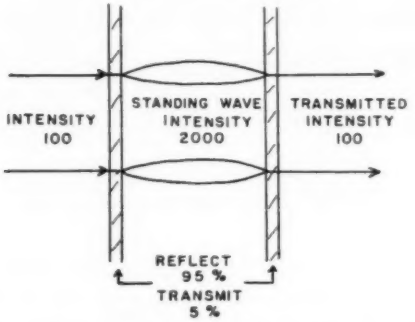

FIG. 1—A first-order interference filter showing the incident, standing, and transmitted waves for a wave-length at the peak of the passband.

wave of exactly the same phase. Eventually, the standing wave reaches an intensity of 2000; since each reflecting layer transmits 5 per cent., a wave of intensity 100 is transmitted to the outside on each side of the filter. The wave on the left turns out to be exactly out of phase with the reflected wave, and they cancel each other. Since no light can be reflected and there is no absorption, it must all be transmitted.

Now consider a neighbouring wave-length for which the thickness of the spacer is not quite $\lambda/2$. A standing wave can still build up, but it cannot become as intense because after a few reflections the waves begin to get out of phase. Suppose the wave-length is such that the standing wave reaches an intensity of 1000; a wave of intensity 50 leaks back and cancels half of the reflected wave, and the other half is transmitted. Finally, for a considerably different wave-length nearly all the light is reflected and about 0.07 per cent. transmitted. If it were not for the co-operation between the two reflecting layers the transmitted intensity would be 5 per cent. of 5 per cent. or 0.25 per cent. at all wave-lengths; it is actually much higher at the peak and somewhat lower away from the peak.

The filter will pass other wave-lengths for which the spacing is an integral number of half-waves, since the condition for a standing wave is satisfied for them too. The number is called the "order of interference" and is usually 2 for interference filters, though 1 is sometimes used.

So far we have assumed that the partially-reflecting films absorb no light, but this is never true in practice especially for films of metal. Even a small absorption has a large effect on the peak transmission of the filter, partly because of the smaller amplitude of the standing wave,

and partly because the fraction of this that leaks back to cancel the reflected wave is less. As an example, consider a filter like the one just discussed except that each film transmits 4 per cent. and absorbs 1 per cent., still reflecting 95 per cent. The peak transmission is reduced from 100 to 64 per cent.

The transmission of a few representative filters is plotted against wave-length in figure 2. The shape of these curves is such that there are no

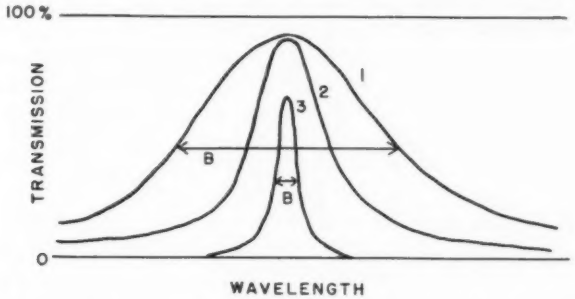

FIG. 2—Sketch of the transmission of three filters as a function of wave-length; 1. low reflectivity layers with no absorption, 2. medium reflectivity, no absorption, 3. high reflectivity, some absorption.

sharp limits to the passband. It is convenient to be able to specify a single "bandwidth" B to characterize a given filter, and the following arbitrary definition is usually adopted: the bandwidth ist he difference of wave-length between the two points at which the transmission has half its peak value. It must always be remembered that the transmission is still large over a considerably wider range of wave-length. Figure 2 shows, as would be expected from the previous discussion, that the bandwidth decreases as the reflectivity of the layers increases. Unfortunately, the absorption always increases with the reflectivity, and the filters of very narrow bandwidth therefore have a low peak transmission. A higher order of interference also gives a narrower band without loss of transmission, but the higher the order the closer are the next passbands which may have to be removed by another filter. For example, a filter may be of the hundredth order at 6000 A.; the bands are about 60 A. apart and may be 1 A. wide.

We must now consider what happens when light is not incident at right angles, since all physical light beams include a range of angles. (An exception to this is the light from a single star, for all practical purposes, but even here one is often interested in a whole field of stars at

once.) It would seem at first sight that the length of the standing wave would increase when it must fit in at an angle, but this is incorrect. This is one case where it is easy to be led astray by considering light as consisting of rays rathcr than waves. While a correct ray picture can be made, the wave constreution of figure 3 gives a better physical idea of the process. Here the peaks of the waves are shown as they appear at a certain instant in a second-order filter. The wave-length has been chosen at the peak of the passband for the angle concerned. The standing wave is no longer standing quite still, but progresses slowly upwards. But the relations are still correct for a wave to leak back and cancel the reflected wave on the left; it is shown as a dotted line. The relation between the wave-length and the filter thickness is shown by the triangle

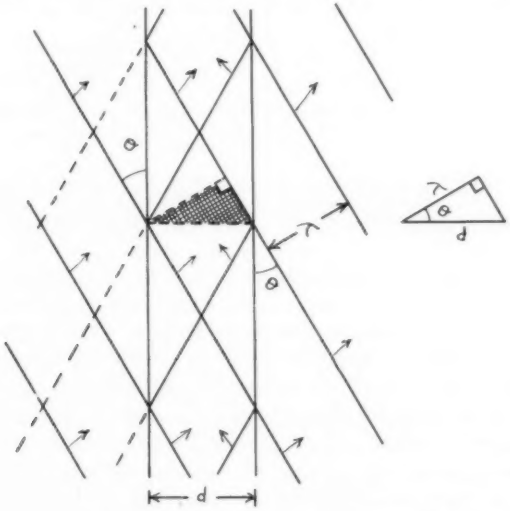

FIG. 3—A second-order filter with light incident at angle θ; the lines are wave-fronts and the rays are parallel to the arrows. The hatched triangle, reproduced at the right, gives the relation between wave-length λ and spacing d; note that λ is less than d.

at the right, and it is seen that the wave-length is shorter than before, not longer. From the triangle it follows that $\lambda = d \cos \theta$; but λ_o, the peak wave-length for normal incidence, is equal to d for this second-order filter. Therefore $\lambda = \lambda_o \cos \theta$, and this is true for any order. With metallic reflecting surfaces the correct equation is more complicated, but this one is a good approximation to it as long as the angle is small.

To this approximation, the passband shifts to shorter wave-lengths without change of shape as the filter is tilted. If it is used in a cone of light, the passband averaged over the cone is broadened as well as shifted; as long as the broadening is small compared with the original bandwidth it may be tolerated. For example, most filters may be used near the focus of a telescope objective lens, but usually not in the corresponding position of a camera lens or Schmidt system. Roughly speaking, a shift of 10 A. is produced by a tilt of 6°, and the shift is proportional to the square of the angle. The exact figures depend on the wave-length and on the materials of which the filter is made, because the angle θ must be measured inside the filter; as a result of refraction, it will be less than the angle of incidence outside.

Materials. The thin layers in a filter are made by vacuum evaporation, just as with aluminium coatings on telescope mirrors. The glass blank does not have to be optically flat, because each layer faithfully follows the contours of the surface. The first filters used silver for the reflecting layers and magnesium fluoride for the spacer, and this construction is still used when a bandwidth of 100 to 200 A. is narrow enough and a transmission of 30 per cent. can be accepted. The bandwidth can be made smaller by increasing the thickness of the silver films, but the transmission soon becomes unacceptably small. The alternative of increasing the order has been used very little, probably because multilayer filters are more satisfactory; commercial filters are of the first or second order.

A remarkable improvement in both bandwidth and transmission is possible by using "multilayer" reflecting surfaces instead of silver. A multilayer is a stack of transparent layers each a quarter-wave in optical thickness, with low and high refractive indices alternating (figure 4a). At each interface there is a partial reflection, just as there is at the surfaces of a window-pane. The thicknesses of the layers are chosen so that all of these partial reflections are in phase and interfere constructively. Thus, a 5-layer stack reflects 89 per cent. and a 7-layer 96 per cent.; it is not at all difficult to reach 99 per cent. by adding more layers. While the quarter-wave condition holds for only a single wave-length, it is near enough over a range of about 25 per cent. above and below this; for example, a stack made for 6000 A. reflects well from 5000 to 7000 A. and transmits other wave-lengths almost like uncoated glass. It is the use of these multilayers that has made possible the recent application of the Fabry-Perot interferometer as a highly efficient spectrometer in the way discussed in an earlier article (this JOURNAL, vol. 53, p. 204,

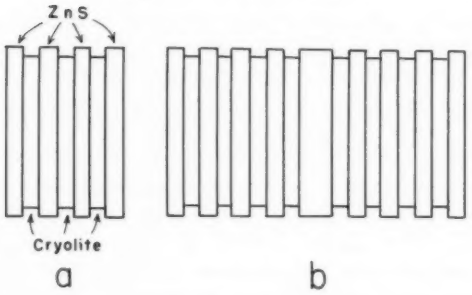

FIG. 4—(a) A 7-layer multilayer reflector. High-index layers are conventionally drawn wider than low to distinguish them. Each layer has an optical thickness of one quarter wave-length.

(b) A 17-layer first-order interference filter. The glass support is not shown in either diagram.

1959); usually a 5-layer stack on each plate is used. Since the multilayer is made of highly transparent materials, the absorption is very near zero, but there is another effect which has the same result in a filter. Light may be scattered into the wrong direction by irregularities and lost to the standing wave just as much as if it had been absorbed. Even so, the "absorption" is much smaller than for a silver layer: well under 1 per cent. for a 7-layer stack and appreciable only when the reflectivity approaches very close to 100 per cent.

The materials used in multilayers are normally zinc sulfide for the high index (2.4) and magnesium fluoride or cryolite for the low (1.35). Cryolite, or sodium aluminium fluoride, is often preferred because it is easier to evaporate. It is not used for anti-reflection coatings on lenses because it is not as durable as magnesium fluoride, but this is immaterial in a multilayer which is fragile in any case because of its greater thickness. At the blue end of the spectrum and especially in the ultraviolet, zinc sulfide is slightly absorbing and must be replaced by something else; antimony trioxide and lead chloride have been used.

A typical first-order multilayer interference filter is shown in figure 4b. It consists of two 8-layer stacks separated by a half-wave spacer, and is therefore a 17-layer filter. The bandwidth is about 30 A. and the peak transmission 80 per cent. The wave-lengths outside the region reflected by the multilayers must be removed by glass or gelatin filters unless the detector is insensitive to them.

The bandwidth can be decreased by using more layers; thus, a 21-layer filter has a bandwidth of 10 to 15 A. and a transmission of about 25 per

cent. Attempts to decrease the bandwidth still further have failed; apparently the irregularities in the layers and interfaces are of this order of size, which is only a few times larger than the molecules themselves. Increasing the order by using a thicker evaporated spacer gives no better results. If a narrower bandwidth is required it may be attained by using a spacer of about the hundredth order made by cleaving mica into very thin sheets, around 0.001 inch or 25 microns thick. If a 7-layer stack is evaporated on each side, a filter results with a band-width of about 1 A. Passbands of neighbouring orders may be removed by an ordinary multilayer filter. Ordinarily the mica will not have just the right thickness to give a passband at the desired wave-length; this is easily cured by making the first layer on one side a little thicker than normal. The extra thickness effectively builds up the mica by the required amount.

Production. We shall describe briefly the methods in use at Saskatoon for producing multilayer and mica filters. They are not claimed to be the only methods possible, and may not be the best possible; in particular, the thickness of the films may be measured in several different ways. Silver filters require a different measuring technique which we shall not consider.

The first requirement is a good vacuum system; the vacuum needed is much better than for aluminizing, and the vapour sources produce a much heavier load on the pumps by heating everything around them. A 12-inch bell-jar can be used for filters up to 4 inches diameter, but a larger one is preferable. A big, fast diffusion pump is essential and a liquid-air trap highly desirable; the diameter of the pump should be a large fraction of the diameter of the bell-jar. Some workers recommend the use of a mercury pump, but oil seems to work as long as it is organic, not silicone. Probably the liquid-air trap is the important thing rather than the choice of oil or mercury. Two heavy-current leads are needed for heating the sources, and two high-voltage leads for the cleaning discharge. Shaft seals provide for moving a shutter over the source and for rotating the filter-blank. Much valuable information on vacuum systems of this type is contained in the book by Holland (1956).

A serious difficulty with narrow-band filters is to keep the passband the same over the whole area. Consider a 10 A. bandwidth at 6000 A.; a variation in layer thickness of only 0.1 per cent. will shift the wave-length 6 A., which is more than can be tolerated. If the vapour sources are 10 inches from the blank, a much larger variation will be found over a filter as small as 2 inches across. The remedy is to offset the sources well to the side and rotate the filter rapidly. The rotation ensures that the

layers will be symmetrical about the axis of rotation; if the sources are offset at the proper angle they will also be very uniform to a considerable distance from the axis. The particular angle is critical and depends on the source and its orientation; its value is about 45° but it must be found by trial and error. If the angle is to be well-defined and reproducible the source must be fairly small and rigid; this requirement seems to disqualify the simplest type in which the material to be evaporated is heated from above by radiation from a red-hot spiral of tungsten wire. Our sources are made by rolling a sheet of 0.002-inch molybdenum into a cylinder with a slot along its length, about 1 inch long and 0.3 inch diameter. The material is put inside and the "boat" heated by passing an electrical current of about 100 amperes through it.

The bearing on which the filter rotates is hollow so that a beam of light can be passed through to measure the thickness of the layers. The light is monochromatic of the wave-length which the filter is to pass; a photomultiplier tube and amplifier measure variations in the transmitted intensity with high sensitivity. A filter normally starts with a layer of zinc sulfide which increases the reflection and therefore reduces the transmission; when the latter goes to a minimum the layer has the right thickness. One cannot be sure of a minimum until it is past, but with care and the use of a sensitive indicator, this trouble is not serious. The second layer, of cryolite, increases the transmission and evaporation must stop at a maximum; the third layer stops at a minimum; and so on. For the spacer, if zinc sulfide is used, the thickness of half a wave is determined by going past the minimum and stopping at the following maximum instead. This method allows the passband to be placed within about 10 A. of the desired wave-length.

A mica filter must be made in two stages, one for each side. The first is made in just the same way as a low-order filter. For the second, the layers must be measured on a separate piece of glass at the same distance from the sources, since they cannot be measured on the filter itself.

Applications. The filters find considerable use in the laboratory; typical applications are isolation of single spectral lines and production of almost monochromatic light for absorption measurements of solutions. We shall be concerned with "astrophysical" uses in a broad sense which includes such things as aurora and airglow; usually the light is faint and often the bandwidth must be very narrow.

Photography of gaseous nebulae in the light of individual spectral lines has interesting possibilities. In the first place, the forms may be different for different lines; this is well known for the Crab Nebula, for instance.

Second, it is often possible to get a denser image of the nebula with a filter than without, because a much longer exposure can be used before sky fog sets in. A description of experiments along this line is given by Ring (1956).

Strömgren (1956) has discussed a method of measuring the spectral class of a star by photoelectric photometry through a number of carefully selected interference and ordinary filters. The measurements require only 8 minutes per star, much less than the time needed to expose a spectrogram. The absolute magnitude of the star can be found in this way with very satisfactory accuracy.

Applications of the mica filter with its bandwidth of 1 or 2 A. have not been made yet, but it may turn out very useful. Until now, only the Lyot polarizing filter has had as narrow a band, and its very low transmission has restricted it to solar work. The mica filter should work just as well for observing prominences and the like, and should be much cheaper. But its high transmission may qualify it for certain uses in stellar and nebular astronomy as well.

The study of the airglow (light emitted by the upper atmosphere) has always been difficult because of its faintness and because of the background of astronomical light. It has been possible to study it photoelectrically by selecting some of the emission lines or bands with interference filters. Such observations, by Roach, Barbier, and others, have disclosed curious variations in space and time whose origin is not understood but which must be explained by any theory of the airglow. Recently they have made possible the discovery of a curious phenomenon at middle latitudes resembling an auroral arc but emitting only red light which is not intense enough to be visible.

Rapid and complicated colour changes are often apparent in bright aurorae, and they must be caused by changing intensities of various lines and bands. The only way such changes can be measured is by photoelectric photometry of several wave-lengths simultaneously. Interference filters are often used to select the desired regions. Interesting work along these lines has been done by Harang and Omholt, but many possibilities remain.

There are times when even the narrowest-band interference filter is unable to discriminate sufficiently against background light. For example, one may wish to measure aurora in the presence of moonlight, airglow in moonlight or starlight, or sodium and other lines in twilight. It is possible to discriminate against the continuous background by making use of a "wedge" interference filter. This is made by mounting the filter in the vacuum system with one end a little lower than the other; the

layers deposited at this end are then a little thicker since they are nearer the sources, and they transmit a longer wave-length. The wedge is a kind of spectroscope for a very limited spectral region. The spectral line which one wishes to detect passes through a single strip across the filter, and the background is transmitted uniformly along the whole filter. A rotating disk can then be arranged so that it "chops" the light from the line but not the background. A photomultiplier produces an alternating voltage whose amplitude depends only on the intensity of the line; a background 10 to 100 times more intense can be suppressed. An auroral recorder built according to this principle can measure the faintest visible aurora but ignores bright moonlight and twilight completely. A similar device is used to measure the sodium D lines in twilight. Possibly the same principle would be useful in measuring the intensity of a stellar absorption line such as Hβ, which is measured by a static method in Strömgren's work.

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NOTES FROM OBSERVATORIES

David Dunlap Observatory, Richmond Hill, Ontario

On May 31st, 1960, the Observatory took notice of the twenty-fifth anniversary of its opening. The staff and a few guests, including Mrs. Moffat Dunlap and her son, Moffat Dunlap, and Dr. R. K. Young, Director Emeritus, enjoyed a special showing at the Richmond Hill Theatre of the award-winning National Film Board's "Universe", which had been filmed in part at the Observatory several years ago. The group then went to the Observatory for a buffet supper and a showing of old films of the opening of the Observatory by the late Mrs. Jessie Donalda Dunlap in 1935 and of the laying of the cornerstone by the late David Moffat Dunlap in 1932.

The wet and cloudy spring has made for a poor observing season in the first half of 1960. However, the primary of the 74-inch telescope was re-aluminized in June and the modified photoelectric spectrophotometer is now ready for testing, so that the visual astronomy equipment is well prepared to take advantage of the summer weather.

Good progress is reported from the radio astronomy group on the projects under way here (see this Journal, vol. 54, p. 86), and plans have been completed for the University of Toronto Building to be built soon at the Algonquin Park Radio Observatory.

Members of the Observatory staff have attended meetings of the National Committee for the I.A.U. in Ottawa (in November 1959) and at Penticton (in June 1960, on the occasion of the opening of the Dominion Radio Astrophysical Observatory) and meetings of the American Astronomical Society in Cleveland (in December) and Pittsburgh (in April). Dr. Helen Hogg attended the dedication in March of the Kitt Peak National Observatory in Arizona. In June the writer visited the Dominion Astrophysical Observatory and Mount Wilson Observatory to confer with astronomers there on the design of the new all-reflection spectrograph to be built for the 74-inch telescope.

At the June meeting of the Royal Society of Canada at Queen's University, Dr. Helen Hogg was elected President of Section III (Physical Sciences) and presided at the "Symposium on Space" of which she was one of the organizers. Dr. MacRae, at the same meeting, gave an invited paper on "Radio Astronomy in Canada".

Dr. Searle has left on leave-of-absence to accept a one-year appointment as Senior Research Fellow in Astronomy at the Mount Wilson and

Palomar Observatories. Dr. K. O. Wright of the Dominion Astrophysical Observatory will replace him for the session.

Five students were graduated in the astronomy course at the University of Toronto in June. One of these, C. R. Purton, was the winner of the Society's Gold Medal and was awarded a National Research Council Bursary as was also another of the group, J. R. Booker. Both of these students have enrolled for graduate work in astronomy here. Other awards to graduate students included a National Research Council Studentship to D. E. Hogg, and the C. A. Chant Graduate Fellowship in Astronomy to K. A. Innanen.

Students employed this summer as summer assistants at the Observatory and for the radio astronomy project in the Department of Electrical Engineering include the following: Judy Bancroft, John Booker, Lorne Braun, Blake Cherrington, Leonard Chow, William Greig, Richard Henry, David Hogg, John Knight, Duncan McNeill, David MacRae, Helge Mairo, Susan Priddle, Christopher Purton, David Sher, Robert Vicary.

Among recent visitors to the Observatory were Dr. G. de Vaucouleurs of Harvard College Observatory, Dr. Robert Hardie of Vanderbilt Observatory, Dr. J. A. Roberts of the California Institute of Technology and Dr. Alan F. Cook of Harvard College Observatory.

JOHN F. HEARD

METEOR NEWS

By Peter M. Millman

National Research Council, Ottawa, Ontario

The Auroral Green Line in a Leonid Spectrum

The forbidden line of neutral oxygen at 5577 A., often termed the auroral green line, was first found in three Canadian meteor spectra by Halliday (1958). Since then it has been identified in a number of additional spectra (Halliday 1960, Millman 1959, Russell 1959). Evidence for the appearance of the line has also been found in a Leonid spectrum, photographed in 1933 on a programme conducted by the writer at the Harvard Observatory. This is spectrum No. 29 on the world list (Millman 1934). Two faint photographic spectra of the same meteor were secured—one at Blue Hill on an ortho plate where the camera was operated by E. M. Cook, and the second at Oak Ridge on a panchromatic plate exposed by the writer. The Blue Hill photograph is reproduced in figure 1.

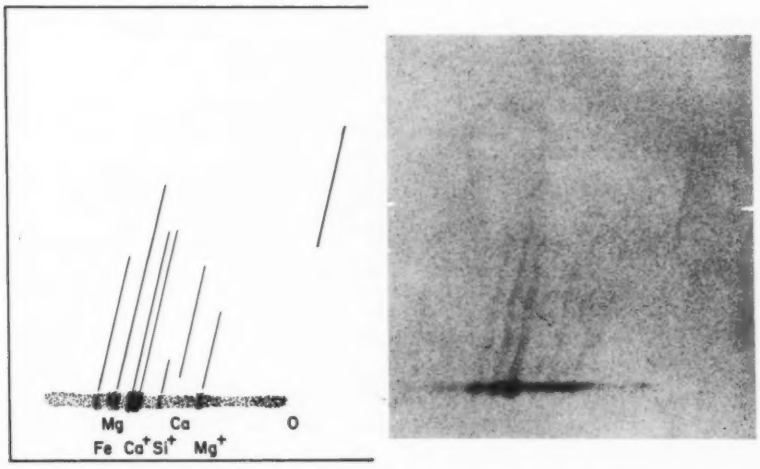

Fig. 1—Spectrum No. 29, a Leonid Meteor photographed on November 17, 1933, at Blue Hill, Mass., 42° 13′N., 71° 07′W.

The meteor appeared at $09^h\ 48^m\ 06^s$ U.T. Nov. 17, 1933 and was visually observed from three stations, apparent visual magnitude being estimated in the range +1 to −3. It was described as having a greenish colour with a terminal burst and a persistent train of approximately 5 sec. duration. The Oak Ridge spectrum is too faint for reliable measurement but the Blue Hill spectrum, though not in perfect focus, shows a detail of 18 lines. The oxygen line appears high and well removed from the rest of the spectrum. Although this feature was noted previously it was not originally measured as its association with the rest of the spectrum seemed doubtful. However, with the more recent evidence from much better spectra studied by Halliday, there seems no doubt that the green line was actually photographed in this very early meteor spectrum.

Spectrum No. 29 has now been re-measured and the wave-lengths, together with the multiplet identification on Mrs. Sitterly's system (Moore 1945), are listed in Table I.

TABLE I

MEASURED WAVE-LENGTHS IN SPECTRUM No. 29

λ	Intensity	Multiplets Identified
3744	medium	Fe I (5) (21)
3844	medium	Fe I (4) (20) Mg I (3)
3933	strong	Ca II (1)
3969	strong	Ca II (1)
4059	weak	Fe I (43)
4129	medium	Si II (3)
4227	medium	Ca I (2)
4319	weak	Fe I (42)
4383	medium	Fe I (2) (41)
4419	weak	Fe I (2) (41)
4481	strong	Fe I (2) Mg II (4)
4554	weak	Fe II (37) (38)
4651	weak	Fe I (38) Fe II (37) (38)
5060	v. weak	Si II (5)
5590	weak	O I (3F)

The listed wave-lengths for the two lines in the green are approximate only as these are too far from the rest of the spectrum for good calibration to be possible. It will be noted that the oxygen line appears first and disappears early on the trail. The ionized elements are relatively

stronger at the terminal burst which is normal behaviour for spectra of fast meteors.

A good height determination was made difficult by the poor quality of the Oak Ridge photograph and some uncertainties in timing as a result of the watches being erratic in the cold weather. Heights were determined on the basis of both the photography and a number of visual plots from each of three stations. The adopted mean values for heights above sea level are:

oxygen line, beginning 132 km.
oxygen line, end 111 km. terminal burst 101 km.

The uncertainty in these values is two or three kilometres. There is no doubt, however, that the oxygen line appeared relatively high in the atmosphere compared to meteor spectra in general.

In addition to the 12 spectra mentioned by Halliday (1960) in his second paper the green line has been identified in two more photographed by Russell and in two secured at the Meanook-Newbrook Observatories in 1959. Adding the Leonid spectrum described above this makes a grand total of 17 meteor spectra in which the oxygen green line has been found. These spectra are divided among the fast meteor showers as shown in Table II.

TABLE II

METEOR SPECTRA WITH THE FORBIDDEN OXYGEN LINE

Shower	Vel. (km./sec.)	Serial No. on World List
Leonid	72	29
Orionid	66	301, 302, 304, 305
Perseid	60	121, 186, 198, 203, 204, 275, 282 294, (1959 Aug. 11, Aug. 12, Aug. 13)
Lyrid	48	215

In 6 of the 17 spectra the green line is the strongest feature and in 8 others it is the strongest feature in the early part of the trail. Average beginning and end heights for the green line are 112 and 98 km. respectively, while the mean height of maximum light for the corresponding meteors is 90 km.

Mr. R. J. Madill has kindly supplied me with geomagnetic information from the Dominion Observatory. The magnetic K index was zero

when Spectrum No. 29 was photographed, so this supports Halliday's suggested correlation of the height of the green line with the K index. However the sun-spot number was also zero for some considerable period both before and after Nov. 17, 1933. Hence it is evident that even moderate solar activity is not necessary for the green line to appear in meteor spectra.

I have noted that bright meteors of the fast showers frequently appear to the eye as illustrated in figure 2 and this same appearance has been

FIG. 2—The common visual appearance of a bright meteor belonging to a high velocity shower.

called to my attention by other visual meteor observers. Halliday suggests that the average duration of the green line is about 1 sec. and may account for the short duration visual trains. This would nicely explain the form illustrated in figure 2 as the one-second green line train would lie along the upper part of the trail and would still be visible when the head of the meteor was passing through its position of maximum light.

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VARIABLE STAR NOTES

By Margaret W. Mayall

The American Association of Variable Star Observers, Cambridge, Mass.

154428 *R Coronae Borealis.* The discovery of R Coronae Borealis and R Scuti, two of the most interesting and best observed irregular variable stars, was announced by Edward Pigott in 1795. Mr. Pigott (1768–1807), who had also discovered the variability of the Cepheid η Aquilae in 1784 and had determined its period of 7.2 days, published one of the first variable star catalogues in 1786. In it he listed 8 variables and 4 novae—all that were known at that time. Pigott lived in York, England, and was a close friend and observing colleague of John G. Goodricke (1765–1786), who in the course of his short life was the first to solve the problem of the cause of the variability of Algol, and who also discovered the variables β Lyrae and δ Cephei.

The accompanying light curve shows that R CrB has been almost continuously observed since 1843. Earlier observations by Pigott showed it nearly constant at 6th magnitude from 1783 to 1785, and not visible in 1795 and 1796, with the exception of two brief maxima.

Prior to 1890, most of the other observations were made by Koch, Olbers, Argelander, Heis, Baxendell, Schmidt, Schönfeld, Winnecke, Kuhn, Auwers, Hartwig, Schwab, Sawyer, Safarik, Gore and Wilsing. After 1890, the light curve depends on observations made at the Harvard Observatory and later by members of the American, British, and French variable star associations. The curve from 1843 to 1923 was compiled by Leon Campbell, and was published in *Harvard Circular* 247.

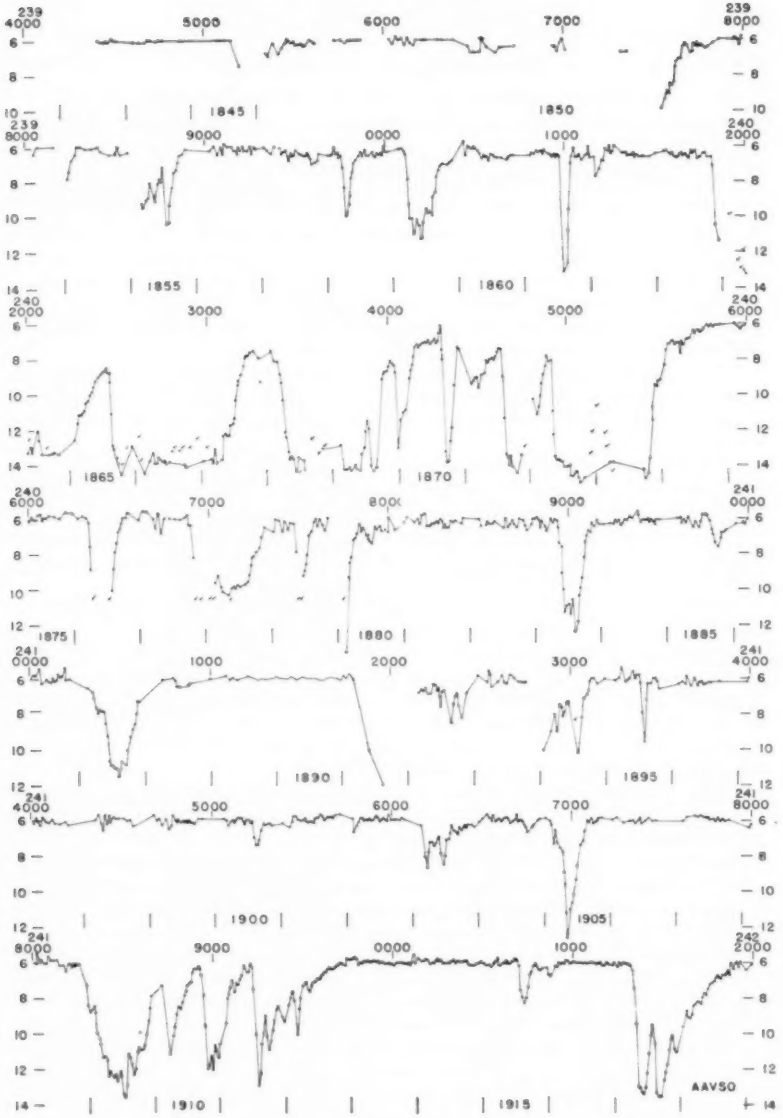
R CrB has been called the "ideal" irregular variable, with times of minimum distributed absolutely at random, according to the laws of pure chance.

Any attempt to define the type of variations must necessarily be vague. The star usually has a sharp drop to minimum and a slow rise back to normal brightness, but an inspection of the curve reveals many slow declines, and also many rapid rises. It remained below normal for about 10 years following the early 1860's. Conversely, it stayed constant at maximum magnitude from 1924 to 1934. The 1960 minimum is the first good decrease R CrB has had since 1952.

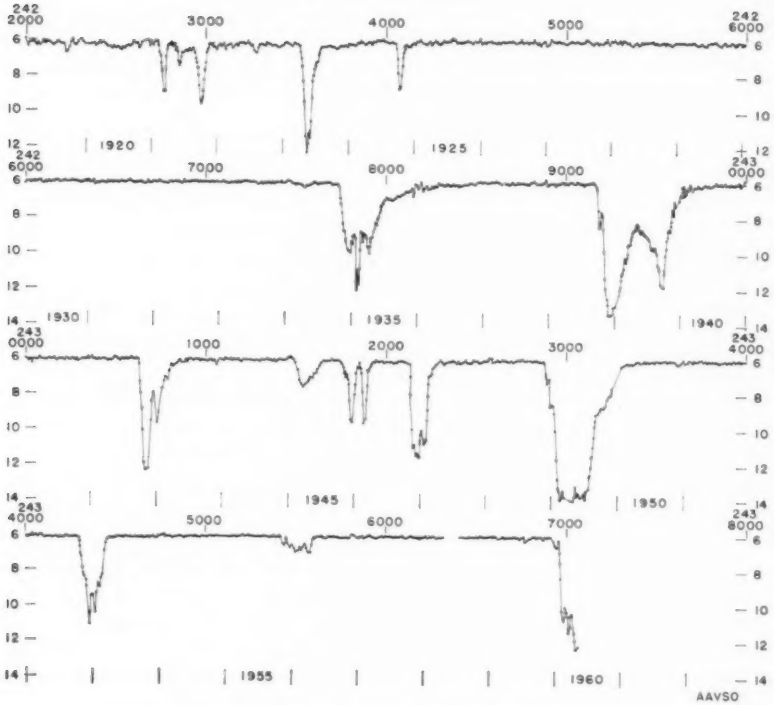
185213 *Nova Herculis 1960.* A nova discovered by visual means is a rare occurrence. The last one was Nova (CP) Puppis, discovered by Bernard H. Dawson, of La Plata, Argentina, on November 8, 1942.

Nova Herculis was discovered by Olaf Hassel, of Årvoll, Norway, on March 7, 1960. After the discovery was announced, Honda, in Japan, found the nova on photographs. On Feb. 27, it was not seen, and fainter than 10; on March 4, it was 3.0; March 5, 3.5; and on March 7, it was 4.4. The nova was also identified by Edwin Weston and Leonard Solomon on two photographs taken with the Baker-Nunn satellite tracking camera in Argentina: photovisual magnitude 2.8 on March 4, and 3.9 on March 5. The decrease after the first rapid drop of 2 magnitudes, has been at the rate of one magnitude in 20 days. It was a little fainter than 8 by the middle of May.

Mr. Hassel works in the climate section of the Norwegian Meteorological Institute, and has been an active amateur astronomer for many years. He was one of the discoverers of Comet Jurlow Achmarov Hassel, on April 16, 1939. He is a member of the Norwegian Nova Society, and among his other independent discoveries may be listed: Brorsen-Metcalf Comet, Sept. 2, 1919; Nova (V476) Cygni, Aug. 24, 1920; and Reid's Comet, April 26, 1921.



117 years of R Coronae Borealis 154428 (*continued next page*)



117 years of R Coronae Borealis 154428

When asked for details about his discovery of Nova Herculis, he wrote: "On the 7th of March, at 0520 M.E.T., I hurried up to the loft and opened up the sky-light. Quite sub-consciously I set the prismatic telescope directly at Nova Herculis 1960, which I was unable to see with the naked eye. Through the telescope I immediately saw a star of the 5th magnitude, which I had not previously seen in the course of my many years' search for Novae in the Milky Way. This latest Nova discovery of mine is quite as fantastic as the occasion in 1934, when for 10 minutes I observed Nova Herculis through a break in the clouds, on Christmas Eve. Apart from those 10 minutes I had been unable to see any stars at all from Dec. 1st until New Year's Eve. All through December the weather had been cloudy with precipitation."

A.A.V.S.O. Nova Search Report (From George Diedrich, Chairman): Observations of Nova Search Areas were made by the following 25 observers for a total of 405 area-nights reported. Each name is followed by the number of observations made in January and then February, 1960: R. Bishop—8, 0; B. Brown—6, 0; F. J. DeKinder—16, 2; D. Diedrich—6, 3; G. Diedrich—9, 4; C. L. Drolet—21, 14; P. Evans—0, 17; K. Fuller—10, 7; G. Gaherty, Jr.—10, 4; B. Green—8, 0; D. Humphrey—0, 9; W. Isherwood, Jr.—17, 18; D. Matthies—0, 3; H. Nightingale—9, 13; D. W. Orchiston—14, 19; C. J. Phillips—34, 20; R. Price—0, 9; B. Reber—2, 0; G. W. Smith—0, 3; F. Traynor—12, 6;

R. Walk—8, 5; W. A. Warren—10, 8; Geo. Wedge—4, 3; I. K. Williamson—3, 3; K. Zorgo—18, 10.

Observations received during March and April 1960: A total of 7,375 observations was received for the two months: 3,265 from 60 observers in March, and 4,110 from 83 observers in April.

Observer	March		April		Observer	March		April	
	Var.	Ests.	Var.	Ests.		Var.	Ests.	Var.	Ests.
Adams, R. M.	63	123	81	163	Lynch, J. S.	1	1	2	2
Anderson, C. E.	21	40	25	25	Maran, S. P.	..	..	1	5
Anderson, E.	..	..	5	13	McLaughlin, D. B.	..	..	1	2
Axelson, K.	..	..	4	7	McPherson, C. A.	12	12	22	24
Barroso, J.	..	..	3	5	Mende, R.	..	..	1	11
Berg, J.	38	47	21	42	Miller, R. W.	..	..	42	120
Bicknell, R. H.	12	37	13	40	Miller, W. A.	10	15	6	6
Brady, R. F.	32	65	31	81	Montague, A. C.	9	9	26	40
Braune, W.	19	93	33	136	Morgan, F. P.	16	23	21	42
Breckinridge, J.	7	7	..	..	Mourilhe S., P.	..	..	3	3
Brinley, F. J.	3	3	..	..	Muniz B., L.	..	..	5	9
Cragg, T. A.	208	269	101	106	Nightingale, H. C.	..	..	11	17
Crotty, J.	2	2	..	..	Olmsted, M.	1	7	2	10
Darsenius, G.	..	..	1	2	Oravec, E. G.	166	273	185	396
de Kock, R. P.	98	453	110	469	Orchiston, D. W.	..	..	34	171
Diedrich, DeL.	1	1	2	2	Pearcy, R. E.	..	..	2	2
Diedrich, G.	1	1	5	6	Peltier, L. C.	15	24	26	129
Dudley, R. R.	13	22	5	5	Price, R. T.	..	..	13	45
Engelkemeir, D.	9	23	9	22	Quester, W.	6	18	9	35
Erpenstein, O. M.	12	24	10	18	Renner, C. J.	40	40	..	..
Estremadoyro, G.	..	..	8	10	Ricker, C. L.	..	..	2	2
Estremadoyro, V.	..	..	4	5	Rizzo, P. V.	8	15	11	13
Evans, P.	..	..	24	151	Robinson, J. C.	..	..	8	10
Farmer, R.	..	..	2	2	Robinson, L. J.	55	231	35	128
Fernald, C. F.	212	301	148	249	Rosebrugh, D. W.	16	95	12	57
Fletcher, D. M.	..	..	6	6	Rudolph, R.	23	57	..	..
Ford, C. B.	89	92	24	32	Schell, C.	2	2	..	..
Friton, E. E.	1	1	..	..	Schiff, S.	2	2	..	..
Fuller, K.	15	15	35	36	Segers, C. L.	..	..	59	202
Giffen, C. H.	..	..	14	20	Severin, M. A.	..	..	14	71
Glenn, F. R.	..	..	1	1	Sharpless, A. P.	19	20	21	21
Glenn, W. H.	..	..	1	3	Shinkfield, R. C.	..	..	21	76
Godfrey, N. B.	33	33	..	..	Smith, G. W.	5	5	11	11
Grossenbacher, R.	14	14	9	9	Smith, J. R.	3	4	20	20
Hartmann, F.	142	146	127	130	Solomon, L.	..	..	1	4
Hegyi, L.	3	6	..	..	Stanton, R.	24	24	29	44
Hicks, R. L.	1	5	1	3	Szekely, C.	6	6	..	..
Houston, W. S.	32	43	..	..	Taboada, D.	88	165	56	77
Hull, A.	3	3	1	1	Templeton, J. H.	4	4	3	3
Humphrey, D. R.	..	..	2	2	Thaly, K.	7	37	..	..
Hunter, T.	..	..	1	1	Thomas, M. A.	..	..	1	9
Judd, D.	19	19	49	51	Traynor, F.	..	..	17	61
Kelly, F. J.	12	19	14	17	Valdez, M.	18	18	..	..
Kindt, O. H.	2	2	..	..	van Zyl, L. L.	15	39	16	35
Knowles, J. H.	9	28	7	7	Wend, R. E.	1	1	..	..
Kunicki, J.	1	1	..	..	Wickett, S.	..	..	1	1
Lacchini, G. B.	..	..	12	58	Williamson, L.	..	..	3	15
Lattey, M.	..	..	6	7	Williamson, P.	..	..	2	9
Leithoff, D.	..	..	5	5	Wilson, C. F.	..	..	35	37
Lowder, W. M.	..	..	30	50	Yamada, T.	46	180	43	137

REVIEW OF PUBLICATIONS

The Transits of Venus, A Study of Eighteenth-Century Science by Harry Woolf. Pages xiii+258; 6¼×9½ in. Princeton, Princeton University Press and Toronto, S. J. Reginald Saunders Co., 1959. Price $6.90.

One of the most fascinating chapters in the history of astronomy is that of the enormous effort to observe the transits of Venus in 1761 and 1769, for which expeditions went literally to the ends of the earth. This volume provides an authoritative and readable account of the planning behind these expeditions, and gives detailed discussion of the most important ones. Well illustrated with facsimiles of original documents and pictures, accompanied by an extensive bibliography, the book provides a mine of information on the subject.

The author shows that, despite the famous black-drop effect and other trials, the observations of the transits were successful in determining a better value for the solar parallax than was previously available. He describes also how the interest in instruments for the expeditions, as outlined by Dr. James Bradley, Astronomer Royal, led to development and improvement of telescopes.

An excellent biography of the French astronomer Joseph-Nicolas Delisle, who deserves to be better known, emphasizes the fact that actually it was Delisle who finally precipitated enthusiasm for the transit of Venus expeditions which Halley had urged many years earlier. Canadians will be interested in the instructions sent by Delisle to the Governor General of Canada in 1752 for observing the transit of Mercury May 6, 1753 from Quebec. We learn also that in the 18th century, as now, Russia was enormously interested in building up science at substantial expense, but was then leaning heavily on astronomers from outside its own boundaries.

Because so much of the material is based on the work of French astronomers, passages in this language are widely quoted and a reading knowledge of French is necessary in much of the book. It is unfortunate that the two astronomical diagrams are marred by misprints; that of the transits on page 19 has a confusion of g's and q's, while that of the planetary aspects on page 201 has east and west interchanged. The bibliography does not include the series of papers on the subject in this JOURNAL in 1951. The discussion of the intervals between transits of Venus on page 4 does not indicate that, as centuries pass, these intervals will change.

The author merits our respect for the many hours of delving in European libraries which have brought to light this important and fascinating material, and for the readable form in which he has put his information. This is an excellent reference book for all astronomical libraries, and will be especially good to have on hand for comparison when the next transits occur in 2004 and 2012.

HELEN SAWYER HOGG

Kepler by Max Caspar. Translated and edited by C. Doris Hellman. Pages 401; 6.5×9.5 in. New York, Abelard-Schuman, 1960. Price $7.50.

Max Caspar's *Johannes Kepler* (Stuttgart, 1948) was the fruit of forty years of labour. On its appearance, it was hailed as 'monumental' and 'a classic'. Could it be improved upon? Yes. Dr. Hellman has shown that gold can be refined. She took ten years to do her translation. Her *Kepler* is much more than a translation; it is a new edition of Caspar's work. It is enhanced by valuable footnotes. It is, undoubtedly, the best biography of Kepler ever written in any language. One regrets that it was not more sumptuously produced. The publisher, of course, had a problem; the addition of the footnotes would have led to a very bulky volume if they had not been printed in small type never meant for aged eyes.

M. W. BURKE-GAFFNEY

Principles of Optics by Max Born and Emil Wolf with contributions by A. B. Bhatia, P. C. Clemmow, D. Gabor, A. R. Stokes, A. M. Taylor, P. A. Wayman and W. L. Wilcock. Pages xxv+803, $6\frac{1}{2}\times9\frac{3}{4}$ in. New York, London, Paris, Los Angeles, Pergamon Press, 1959. Price $17.50.

The discussion in this book is restricted to those optical phenomena which may be treated by Maxwell's theory without reference to the molecular structure of matter, that is, it is restricted to phenomena which can be treated by regarding light as waves, not photons, and media as continuous distributions of matter, not configurations of atoms or molecules. Although numerous optical devices are briefly described, the book is primarily of an advanced theoretical nature with great attention to rigorous formal development.

The chapters on electromagnetic theory, geometrical optics (including the theory of aberrations), interference, Fresnel-Kirchhoff diffraction theory and the optics of metals and crystals are basically translations

from Born's classic book "Optik" (1933) with revisions and extensions. The updating of these chapters is considerable but not quite complete; for example, important developments in interferometric spectral analysis (see, for example, *Proceedings of the Bellevue Conference*, 1957, *J. de phys. et radium*, vol. 19, 1958; Connes, J. and Gush, H. P., *J. de phys. et radium*, vol. 20, p. 915, 1959; Kahn, F. D., *Ap. J.*, vol. 129, p. 518, 1959; Mertz, L., *Ap. J.*, vol. 131, p. 519, 1960) are not mentioned in the long chapter (114 pages) on the theory of interference and interferometers. A short chapter describing the main image-forming optical systems provides a framework for those parts of the book which deal with the theory of image formation. There are substantial chapters on the diffraction theory of aberrations, the interference and diffraction of partially coherent light, rigorous diffraction theory and the diffraction of light by ultrasonic waves.

The wealth of comprehensive and up-to-date material in this book makes it a valuable reference work; however, it is too massive for use as a classroom text-book.

E. H. RICHARDSON

Our Sun (revised edition) by Donald H. Menzel. Pages 350; $6\frac{1}{4}\times9\frac{1}{2}$ in. Toronto, S. J. Reginald Saunders and Company Limited, 1959. Price $9.00.

The 1949 edition of this familiar book has been brought up to date and presented in an entirely new and larger format. Major revisions and additions to the text appear in those chapters dealing with sun-spots, spicules and granules, radio emission from the disturbed sun, and solar-terrestrial relations. Two new chapters have been created by expanding and rearranging several sections of the earlier edition, and about one-third of the 200 excellent illustrations are new.

The author has attempted the difficult task of covering the whole field of solar physics for the general reader in a book of limited size. One can hardly expect that all topics would receive the same attention and indeed one finds in both editions that the emphasis dwells heavily on the active sun. This reviewer feels that full justice has not been given to the very substantial work done on the physics of the quiet sun. It is also surprising that some recent important developments have received such scant attention, an obvious example being the measurements from rockets of the solar ultraviolet and X-ray radiations made by the U.S. Naval Research Laboratory.

The book is at its best when the author employs his natural talent for drawing vivid analogies to help explain some complex phenomenon. Specialists will find shortcomings throughout, but it is nevertheless a useful book for the amateur astronomer or the student at an elementary level.

V. GAIZAUSKAS

Standard Handbook for Telescope Making by N. E. Howard. Pages 326; $9\frac{1}{4}\times6\frac{1}{4}$ in. New York, Thomas Y. Crowell Company, 1959. Price $5.95.

Mr. Howard has shown a pleasant freshness of outlook in this book which is intended for the student or beginning amateur. The possessor of this *Handbook* need not hesitate to tackle the production of a worthwhile telescope. The chapters dealing with grinding, polish, pitch laps and Foucault test have a new approach which makes his book interesting reading and results in clarity of understanding.

The word parabolizing frightens many people but Mr. Howard's description makes it a down-to-earth subject, without the complications of mathematical formulae. The mathematics in Chapter 10 on diagonal mirrors will not frighten the most timid student. Chapter 11 touches only lightly on the subject of eyepieces and may not provide the more advanced amateur with the information he requires; however, useful references are included at the end of the chapter.

Although for some readers Mr. Howard may not have given sufficient information regarding mountings and about housing the telescope once it is made, the book provides a useful foundation upon which to start building. However, I do not think lead can be tapped like steel (p. 202) and recommend casting a nut in the weight.

This book can be recommended highly to those with the urge to build their own instrument, but who feel the project may be a little beyond them. The comprehensive glossary and appendix at the end of the book provide invaluable references to up-to-date information.

F. LUNN

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